Disjunctive Termination for Affluent Families

Vincent van Oostrom

Part I: Affluence for Termination

Part II: Disjunctive Termination for Jumping Families

Disjunctive Termination

Modularity idea

condition such that family $(\to)_I$ is terminating iff its union $\bigcup(\to)_I$ is

$(\to)_I$ denotes $(\to_i)_{i \in I}$; family is terminating if every member is

Disjunctive Termination

Modularity idea

condition such that family $(\to)_I$ is terminating iff its union $\bigcup(\to)_I$ is

Example (monster barring)

Disjunctive Termination

Modularity idea

condition such that family $(\rightarrow)_I$ is terminating iff its union $\bigcup(\rightarrow)_I$ is

Example (monster barring)

- both $a \triangleright b$ and $b \blacktriangleright a$ terminating, but union $\rightarrow := \triangleright \cup \blacktriangleright$ is not (feature interaction between steps brought about by composition)

Disjunctive Termination

Modularity idea

condition such that family $(\to)_I$ is terminating iff its union $\bigcup(\to)_I$ is

Example (monster barring)

- both $a \rhd b$ and $b \blacktriangleright a$ terminating, but union $\to := \rhd \cup \blacktriangleright$ is not
- $n \to_n n+1$ terminating for all n, but union $\bigcup(\to)_{\mathbb{N}}$ is not (infinite families must be ruled out)

Disjunctive Termination

Modularity idea

condition such that family $(\to)_I$ is terminating iff its union $\bigcup(\to)_I$ is

Example (monster barring)

- both $a \triangleright b$ and $b \blacktriangleright a$ terminating, but union $\to := \triangleright \cup \blacktriangleright$ is not
- $n \to_n n+1$ terminating for all n, but union $\bigcup(\to)_{\mathbb{N}}$ is not

rest of talk: how to contain feature interaction for finite families (I is finite)

State of the past art on Disjunctive Termination

Theorem

for $\rightarrow := \bigcup \mathcal{F}$ *for finite family* $\mathcal{F} := (\rightarrow)_I$

- *if* $\rightarrow$ *is transitive then* $\mathcal{F}$ *is terminating iff* $\rightarrow$ *is, if* $\#I = 2$ *(Geser 1990)* *(above example not transitive since not reflexive)*

State of the past art on Disjunctive Termination

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- *if* $\to$ *is transitive then* $\mathcal{F}$ *is terminating iff* $\to$ *is, if* $\#I = 2$
- *if* $\to$ *is transitive then* $\mathcal{F}$ *is terminating iff* $\to$ *is*
  (Podelski & Rybalchenko 2004; unaware of Geser's result)

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- *if* $\to$ *is transitive then* $\mathcal{F}$ *is terminating iff* $\to$ *is*

cannot prove latter from former by induction (Steila and Yokoyama 2016)

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cannot prove latter from former by induction

Plan for talk

1. relax transitivity to affluence
2. $\to$ non-terminating $\Rightarrow$ $\to_i$ non-terminating for some i for $\#I = 2$, entails
3. $\to$ non-terminating $\Rightarrow$ $\to_i$ non-terminating for some i for finite I by induction

1 Relaxing transitivity to affluence

Definition (& Zantema 2012)

$\blacktriangleright, \triangleright$ is affluent if $\triangleright \cdot \blacktriangleright \subseteq \triangleright \cup \blacktriangleright$

Example (illustrating richness of affluence)

if $\blacktriangleright = \triangleright$ it expresses transitivity

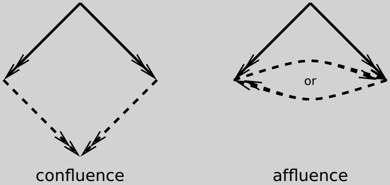
1 Relaxing transitivity to affluence

Definition ()

$\blacktriangleright$, $\triangleright$ is affluent if $\triangleright \cdot \blacktriangleright \subseteq \triangleright \cup \blacktriangleright$

Example (illustrating richness of affluence)

setting $\triangleright := \twoheadleftarrow$ and $\blacktriangleright := \twoheadrightarrow$ affluence is $\twoheadleftarrow \cdot \twoheadrightarrow \subseteq \twoheadleftarrow \cup \twoheadrightarrow$



affluence (flowing toward) strengthens confluence (flowing together)

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Example (illustrating richness of affluence)

setting $\triangleright := \geq$ and $\blacktriangleright := \leq$ affluence expresses totality: $n \geq m$ or $n \leq m$ on $\mathbb{N}$ (because assumption $n \geq \cdot \leq m$ holds for any n, m as $0 \leq n, m$)

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$\blacktriangleright, \triangleright$ is affluent if $\triangleright \cdot \blacktriangleright \subseteq \triangleright \cup \blacktriangleright$

Example (illustrating richness of affluence)

setting $\triangleright := \sqsupseteq$ and $\blacktriangleright := \sqsubseteq$ for $\sqsubseteq$ the prefix order on finite $\rightarrow$-reductions
affluent iff $\rightarrow$ is deterministic
(in the CompCert formalisation)

1 Relaxing transitivity to affluence

Definition ()

$\blacktriangleright, \triangleright$ is affluent if $\triangleright \cdot \blacktriangleright \subseteq \triangleright \cup \blacktriangleright$

Lemma (restriction; key)

let $\to := \blacktriangleright \cup \triangleright$ *for* $\blacktriangleright, \triangleright$ *affluent, and* γ *some* $\to$*-reduction*
then their restriction (to γ*)* $\blacktriangleright\restriction\gamma, \triangleright\restriction\gamma$ *is affluent*

1 Relaxing transitivity to affluence

Definition ()

$\blacktriangleright, \triangleright$ is affluent if $\triangleright \cdot \blacktriangleright \subseteq \triangleright \cup \blacktriangleright$

Intuition

affluence to compress consecutive out-of-order steps gives reduction that is:

- progressive ($\blacktriangleright$-steps occur before $\triangleright$-steps in the reduction)

1 Relaxing transitivity to affluence

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Intuition

affluence to compress consecutive out-of-order steps gives reduction that is:

- progressive ($\blacktriangleright$-steps occur before $\triangleright$-steps in the reduction)
- preferential ($\blacktriangleright$-steps are preferred over $\triangleright$-steps in the reduction)

2 family union non-terminating $\Rightarrow$ member is ($\#I = 2$)

let $\rightarrow := \blacktriangleright \cup \triangleright$ and $\blacktriangleright, \triangleright$ affluent ($\triangleright \cdot \blacktriangleright \subseteq \triangleright \cup \blacktriangleright$)

Theorem

if $\rightarrow$ is non-terminating, then $\blacktriangleright$ or $\triangleright$ is non-terminating

2 family union non-terminating $\Rightarrow$ member is ($\#I = 2$)

Theorem

if $\rightarrow$ is non-terminating, then $\blacktriangleright$ or $\triangleright$ is non-terminating

Proof.

- suppose γ is infinite $\rightarrow$-reduction from a

$\square$

2 family union non-terminating $\Rightarrow$ member is ($\#I = 2$)

Theorem

if $\rightarrow$ is non-terminating, then $\blacktriangleright$ or $\triangleright$ is non-terminating

Proof.

- suppose γ is infinite $\rightarrow$-reduction from *a*
- restriction $\blacktriangleright\restriction\gamma$, $\triangleright\restriction\gamma$ is affluent (by key lemma)

$\square$

2 family union non-terminating $\Rightarrow$ member is ($\#I = 2$)

Theorem

if $\to$ is non-terminating, then $\blacktriangleright$ or $\triangleright$ is non-terminating

Proof.

- suppose γ is infinite $\to$-reduction from *a*
- restriction $\blacktriangleright\restriction\gamma$, $\triangleright\restriction\gamma$ is affluent
- consider maximal $\blacktriangleright\restriction\gamma$-reduction from *a*

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- if infinite, then $\blacktriangleright$ is non-terminating

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Proof.

- suppose γ is infinite $\rightarrow$-reduction from *a*
- restriction $\blacktriangleright\restriction\gamma$, $\triangleright\restriction\gamma$ is affluent
- consider maximal $\blacktriangleright\restriction\gamma$-reduction from *a*
- if infinite, then $\blacktriangleright$ is non-terminating
- otherwise, ends in some $\blacktriangleright\restriction\gamma$-normal form $\hat{b}$ and proceed as in picture:

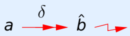
$\square$

2 family union non-terminating $\Rightarrow$ member is ($\#I = 2$)

Theorem

if $\rightarrow$ is non-terminating, then ► or ▷ is non-terminating

Proof.



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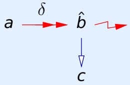

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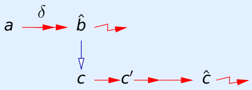

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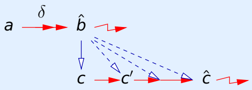

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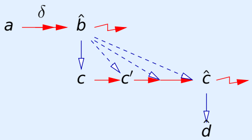

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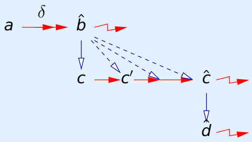

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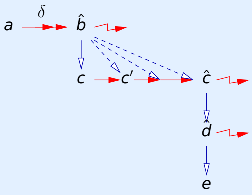

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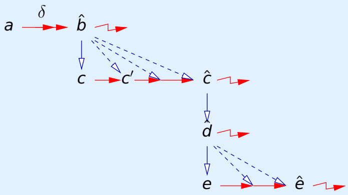

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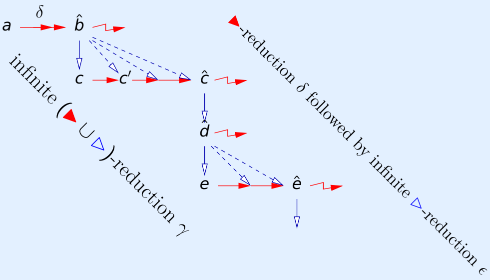

□

2 family union non-terminating $\Rightarrow$ member is ($\# I = 2$)

Theorem

if $\to$ is non-terminating, then $\blacktriangleright$ or $\triangleright$ is non-terminating

Observations

- γ transformed into reduction δ of shape $\blacktriangleright\!\!\blacktriangleright \cdot \triangleright^{\omega}$ or $\blacktriangleright\!\!\blacktriangleright \cdot \triangleright\!\!\triangleright \cdot \blacktriangleright^{\omega}$ progressive (no out-of-order if $\not\infty$) and preferential ($\blacktriangleright$ steps preferred if ∞)

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Theorem

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- $\triangleright$-sub-reductions of δ through objects in $\blacktriangleright\!\restriction\!\gamma$-normal form

3 finite family union non-terminating $\Rightarrow$ member is

Definition

family $\mathcal{F} := (\rightarrow)_I$ is affluent if $\rightarrow_{>i} \cdot \rightarrow_i \subseteq \rightarrow := \bigcup \mathcal{F}$ for all i

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3 finite family union non-terminating $\Rightarrow$ member is

Theorem

if $\rightarrow$ is non-terminating, then $\rightarrow_i$ is non-terminating for some i

Proof by induction on cardinality of I.

- suppose γ is infinite $\rightarrow$-reduction from a
- $\blacktriangleright$, $\triangleright$ is affluent for $\blacktriangleright := \rightarrow_1$ and $\triangleright := \rightarrow_{>1}$ (by assumption)

$\square$

3 finite family union non-terminating $\Rightarrow$ member is

Theorem

if $\rightarrow$ is non-terminating, then $\rightarrow_i$ is non-terminating for some i

Proof by induction on cardinality of I.

- suppose γ is infinite $\rightarrow$-reduction from a
- $\blacktriangleright$, $\triangleright$ is affluent for $\blacktriangleright := \rightarrow_1$ and $\triangleright := \rightarrow_{>1}$
- some reduction δ from a of shape $\blacktriangleright\!\!\blacktriangleright \cdot \triangleright^{\omega}$ or $\blacktriangleright\!\!\blacktriangleright \cdot \triangleright\!\!\triangleright \cdot \blacktriangleright^{\omega}$

$\square$

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- in latter case, $\blacktriangleright$ (*i.e.* $\to_1$) is non-terminating

$\square$

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Proof by induction on cardinality of *I*.

- suppose γ is infinite $\to$-reduction from *a*
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- some reduction δ from *a* of shape $\blacktriangleright\!\!\blacktriangleright \cdot \triangleright^{\omega}$ or $\blacktriangleright\!\!\blacktriangleright \cdot \triangleright\!\!\triangleright \cdot \blacktriangleright^{\omega}$
- in latter case, $\blacktriangleright$ (*i.e.* $\to_1$) is non-terminating
- otherwise, conclude by IH for infinite $\triangleright$-sub-reduction
  (through objects in $\blacktriangleright\!\lceil\gamma$-normal form, so $(\to)_{>1}$ affluent on it)

$\square$

3 finite family union non-terminating $\Rightarrow$ member is

Theorem

if $\rightarrow$ is non-terminating, then $\rightarrow_i$ is non-terminating for some i

Observations

- finite family does follow by induction from doubleton family

3 finite family union non-terminating $\Rightarrow$ member is

Theorem

if $\to$ is non-terminating, then $\to_i$ is non-terminating for some i

Observations

- finite family does follow by induction from doubleton family
- proof the same for transitivity (Geser and P & R) instead of affluence

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Theorem

if $\to$ is non-terminating, then $\to_i$ is non-terminating for some i

Observations

- finite family does follow by induction from doubleton family
- proof the same for transitivity instead of affluence
- may strengthen to yield reduction δ of shape $\twoheadrightarrow_1 \cdot \ldots \cdot \twoheadrightarrow_n \cdot \to_k^\omega$ (see paper) progressive (no out-of-order) and preferential (lower index steps preferred)

Application to program termination

Example (program *P*)

```
while x > 0 and y > 0
if x > y then x,y := y,x else  y := y-1
```

terminating transition relation R?

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terminating transition relation R?

1 $T_1 := \{(\langle x, y\rangle, \langle x', y'\rangle) \mid x > 0 \wedge x > x'\}$, $T_2 := \{(\langle x, y\rangle, \langle x', y'\rangle) \mid y > 0 \wedge y = y' + 1\}$

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2 $(T)_{\{1,2\}}$ terminating, their union T contains R, but $\langle 1,2\rangle\ T_2\ \langle 2,1\rangle\ T_1\ \langle 1,2\rangle$

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2 $(T)_{\{1,2\}}$ terminating, their union T contains R, but $\langle 1, 2\rangle \; T_2 \; \langle 2, 1\rangle \; T_1 \; \langle 1, 2\rangle$
3 refine T_2 into $T_2' := \{(\langle x, y\rangle, \langle x', y'\rangle) \mid y > 0 \wedge y = y' + 1 \wedge x = x'\}$ (x untouched)
4 T' is affluent (even $T_2 \cdot T_1 \subseteq T_1$), disjunctively terminating by theorem

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Example (program P)

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3 refine T_2 into $T_2' := \{(\langle x,y\rangle,\langle x',y'\rangle) \mid y > 0 \wedge y = y'+1 \wedge x = x'\}$ (x untouched)
4 T' is affluent (even $T_2 \cdot T_1 \subseteq T_1$), disjunctively terminating by theorem
5 $R \subseteq T'$ so R is terminating

LICS 2024 Test of Time Award

Transition Invariants, by Andreas Podelski and Andrey Rybalchenko

The paper presents in a very clean and elegant way a general characterization of the validity of liveness properties of programs, like termination and other such properties expressed in temporal logic. This is achieved by employing relations over program states, called *transition invariants*, which contain the transitive closure of the state transition relation defined by the program. The key result is that the absence of infinite executions can be reduced to proving that the transition invariant is a finite union of well-founded relations. The authors show how to use such *disjunctively well-founded* transition invariants to validate temporal properties of concurrent systems. The paper has greatly influenced the development of techniques and tools for proving termination of programs automatically since it nicely combines the use of disjunctive well-foundedness with the construction of an abstraction of the program transition relation, which is the transition invariant. The suitability for automation of the approach has been crucial in its success. In addition, the paper also had a large impact on the design of powerful techniques based on termination analysis to (dis)prove a great variety of temporal properties of programs.

Preprint: [pdf]. DOI: [10.1109/LICS.2004.1319598].

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conscientious eulogy?

could maybe imagine missing Geser's result in 2004, but 20 years later?

Disjunctive Termination for Jumping Families

Definition (Doornbos & von Karger 1998)

$\blacktriangleright, \triangleright$ is jumping if $\triangleright \cdot \blacktriangleright \subseteq \triangleright \cup (\blacktriangleright \cdot \twoheadrightarrow)$, for $\rightarrow := \blacktriangleright \cup \triangleright$

Disjunctive Termination for Jumping Families

Definition ()

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Theorem

if $\to$ is non-terminating, then $\blacktriangleright$ or $\triangleright$ is non-terminating

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Theorem

if $\rightarrow$ is non-terminating, then $\blacktriangleright$ or $\triangleright$ is non-terminating

Proof.

same proof, but for different notion of restriction since can no longer guarantee that intermediate objects in compositions $\blacktriangleright \cdot \twoheadrightarrow$ are on infinite reduction (only that they are along it; see paper) □

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$\blacktriangleright, \triangleright$ is jumping if $\triangleright \cdot \blacktriangleright \subseteq \triangleright \cup (\blacktriangleright \cdot \twoheadrightarrow)$, for $\rightarrow := \blacktriangleright \cup \triangleright$

Theorem

if $\rightarrow$ is non-terminating, then $\blacktriangleright$ or $\triangleright$ is non-terminating

Corollary (Doornbos & von Karger 1998; PbP in & Zantema 2012)

$\rightarrow$ is terminating iff $\blacktriangleright, \triangleright$ are

Disjunctive Termination for Jumping Families

Definition (Dawson & Dershowitz & Goré 2018)

family $\mathcal{F}$ is *jumping* if $\rightarrow_{>i} \cdot \rightarrow_i \subseteq \rightarrow_{>i} \cup (\rightarrow_i \cdot \twoheadrightarrow_{\geq i})$ for $i \in I$

allowing $\twoheadrightarrow$ in composition breaks induction, result

Theorem

if $\mathcal{F}$ is non-terminating, then some family member is

Disjunctive Termination for Jumping Families

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Corollary (Dawson & Dershowitz & Goré 2018)

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- affluence and jumping can be combined (blended families; see paper)
- D & D & G do not cite P & R

Checking blends is interesting / non-trivial

Example

does $\to_{>i} \cdot \to_i \subseteq \to \cup \left(\to_i \cdot \twoheadrightarrow_{\geq i}\right)$ for all i, suffice for disjunctive termination?

difference with jumping in left disjunct ($\to$ instead of $\to_{>i}$)

Conclusions

Disjunctive termination is a concept in computer science, particularly in the field of program termination proving. It involves determining whether a program will finish running or could run indefinitely. This is crucial for ensuring that software behaves predictably. . . .

While the term "affluent families" may not directly relate to disjunctive termination, the underlying principles of ensuring reliable software can be crucial for applications used in various sectors, including finance, healthcare, and education. Reliable software can enhance the quality of life and provide better services to families, regardless of their economic status.

In summary, disjunctive termination is a vital area of study in computer science that helps ensure programs run as intended, which can have broad implications for various sectors, including those serving affluent families.

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duckduckgo, Search Assist, August 2025

Future research

- complexity of family in terms of members (Steila & Yokoyama 2016) (exploit $\twoheadrightarrow_1 \cdot \ldots \cdot \twoheadrightarrow_n \cdot \rightarrow_k^{\omega}$)?

Future research

- complexity?
- blending families?
  (used Hans Zantema’s tool Carpa for rapid prototyping of new blends)

Future research

- complexity?
- blending families?
- incorporate in tools?
  (LICS ToT award; exploit affluence weaker than transitivity, *e.g.* CPO)