Abstract A one rule term rewrite system (TRS) which is $\mathsf{UN}^{\rightarrow}$ but not UN.

An abstract rewrite system is UN ($\mathsf{UN}^{\rightarrow}$) if $n \leftrightarrow^* n'$ ($n \;{}^*\!\!\leftarrow\cdot\rightarrow^*\; n'$) $\Rightarrow n \equiv n'$ for normal forms n and n'. $\mathsf{UN} \Rightarrow \mathsf{UN}^{\rightarrow}$ since ${}^*\!\!\leftarrow\cdot\rightarrow^* \subseteq \leftrightarrow^*$, but $\mathsf{UN}^{\rightarrow} \not\Rightarrow \mathsf{UN}$:

$$n \leftarrow a \rightarrow \underset{\circlearrowleft}{b} \leftarrow a' \rightarrow n'$$

Claim The TRS $\mathcal{R} = \{f(a, f(x, y)) \rightarrow f(x, f(a, b))\}$ is $\mathsf{UN}^{\rightarrow}$ but not UN.
Proof $\mathcal{R}$ is not UN since for any normal form $n \not\equiv a$ (using symmetry):

$$f(n, f(a, b)) \leftarrow f(a, f(n, f(a, b))) \leftarrow f(a, f(a, f(n, t))) \rightarrow f(a, \underset{\circlearrowleft}{f}(a, b))$$

To show that $\mathcal{R}$ is $\mathsf{UN}^{\rightarrow}$ consider $n \;{}^*\!\!\leftarrow\cdot\rightarrow^*\; n'$ for normal forms n and n'. W.l.o.g. we may assume both diverging reductions to be standard and to differ already in their first steps. If one of the contracted redexes would be to the left of or nest the other, it would by standardness persist in either normal form which is impossible. Hence the first steps have overlap, but then (see above) a subterm $f(a, f(a, b))$ is created which will persist in either normal form. $\square$