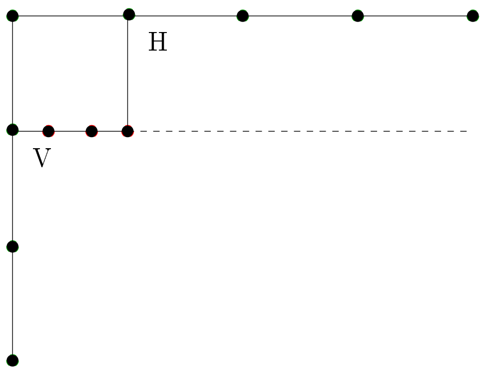

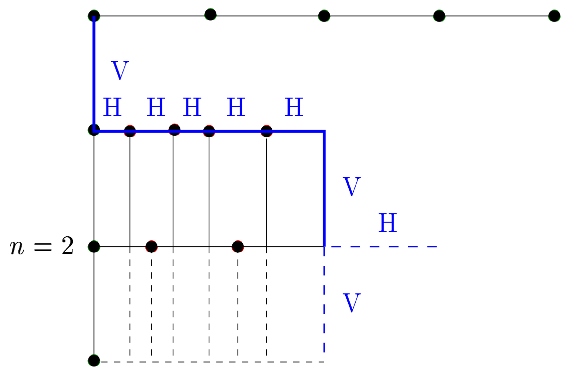

Definition 1 *An* Escher Diagram *(EsD) is an infinite diagram with finite spanning paths.*

First remark that after filling the root corner of an EsD with an elementary diagram as in the figure on the left, at least one of the roots of the areas H and V is the root of an EsD again. If the area H up to the dashed line is infinite this is clear. Otherwise, the area V below the dashed line (this line can only be finite since H is finite) must be infinite. Let's indicate the two possibilities of *moving* from the root of an EsD to the root of another EsD by H(orizontal) and V(ertical). By the argument above sequences of moves can always be extended.

Theorem 2 *Eventually there will be a vertical splitting point on some path from the original root to the root of the subdiagram, for any sequence of moves.*

Proof Consider for a contradiction, an EsD and an infinite sequence of moves not satisfying the condition. We will show that for every prefix σ of the sequence having n vertical moves, it holds that *there's a horizontal path from the endpoint of the nth vertical step in the original EsD to the root of the current EsD*. In the figure on the right, the situation after the sequence VHHHHHV (indicated by the thick blue line) is shown. The claim obviously holds true for the empty prefix ($n = 0$). Suppose that n has been assigned to a prefix σ.

(H) If the next move is H, then the invariant is obviously preserved.

(V) If the next move is V, one observes that all the diagrams to the left of this move must have exactly one vertical step as a conclusion (see the dashed part of the figure on the right), since otherwise a vertical splitting point would have been found.

The procedure ends since no infinite sequence of H-moves is possible due to finiteness of horizontal spanning paths of EsDs, and since V-moves increase the number n which is bounded by finiteness of the vertical spanning path of the original EsD (in the figure n cannot exceed 3). $\square$

Since the argument is symmetric we have as a corollary that every EsD contains a path thru infinitely many horizontal and infinitely many vertical splitting points.