abstract S-terms can eat at most three passive arguments [Vis81, Stelling 12].

Define syntactic classes of *variables*, S*-terms* and *variable tail-terms* in CL.

$$\begin{array}{rcl} \mathcal{X} \ni x & ::= & X \mid x' \\ \mathcal{S} \ni s & ::= & S \mid x \mid ss \\ \mathcal{T} \ni t & ::= & x \mid st \end{array}$$

Using these classes we can state and prove Visser's result. The idea of the proof is that a variable at the tip of the right-spine of an S-term will remain there.

lemma If $sx_1x_2x_3x_4 \to^*_{\mathrm{CL}} s'$, then $s' \equiv s''x_4$, with $x_4 \notin s''$.

proof Remark that $\mathcal{X} \subset \mathcal{T} \subset \mathcal{S}$ and that $\mathcal{X}$ and $\mathcal{S}$ are trivially closed under rewriting. The main observation is that if $s \in \mathcal{T}^i$ and $s \to_{\mathrm{CL}} s'$ by a root-step, then $s' \in \mathcal{T}^i$, for $1 \leq i \leq 4$. This is shown successively for each i, proving that $\mathcal{T}^i$ is closed under rewriting as well and using that for $i = 4$, s cannot be rewritten at the root since $S \notin \mathcal{T}$. To conclude note that $sx_1x_2x_3x_4 \in \mathcal{T}^4$. $\square$

References

[Vis81] A. Visser. Aspects of diagonalization & provability. PhD UU, 1981.